THE AMERICAN MATHEMATICAL MONTHLY.

Entered at the Post-office at Springfield, Missouri, as second-class matter.

VOL. XV. JANUARY, 1908. NO. 1.

NOTES ON THE HISTORY OF THE SLIDE RULE.*

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Few instruments designed for minimizing mental labor in computation offer a more attractive field for historical study than the slide rule. Its development has reached in many directions and has attracted a great variety of intellect. Not only have writers on arithmetic been drawn to it, but also carpenters, excise officers, practical engineers, chemists, physicists, and mathematicians, including even the great Sir Isaac Newton.

And yet, the history of this instrument has been neglected to such an extent that gross inaccuracies occur in standard publications.

The first point I desire to make relates to the invention of the straight-edge slide rule. One of our American manufacturers of slide rules has an instrument on the market, called the "Gunter slide rule" and claims that it "is the original form of the slide rule." As a matter of fact Gunter never invented a slide rule. What Gunter did do was to publish in 1620, six years after Napier's publication of his logarithms, a work containing a description of Gunter's "line of numbers," which, when mounted upon a scale was called "Gunter's scale." On it distances were taken proportional to the logarithms of numbers; it was logarithms laid off upon straight lines. But Gunter's scale contained no sliding parts and, therefore, was not a slide rule.

Charles Hutton, in his *Mathematical Dictionary* (Art. "Gunter's Line"), and also in his *Mathematical Tables*, ascribes the invention of the slide rule to Edmund Wingate, 1627, but he nowhere substantiates his statement by reference to any of Wingate's works. De Morgan in his article "Slide Rule" in the *Penny Cyclopaedia* (1842), and in later publications, ascribes the invention to William Oughtred, a famous writer of mathematical text-books, 1632, and denies that Wingate ever wrote on the slide rule. It will soon appear that De Morgan was ill-informed on this subject, for he had not seen all of Wingate's works, although his criticism of a passage in Ward's *Lives of the Professors of Gresham College* (1740) is well taken. Ward claims that Wingate introduced the slide rule into France

*Read before the Southwest Section of the American Mathematical Society, in St. Louis, November 30, 1907.

in 1624. What he at that time really did introduce was Gunter's scale, as appears from the examination of his book, published in Paris in 1624. To prove or disprove the claim made for Wingate requires the examination of his numerous writings. To the present writer Wingate's publications are not accessible. An inquiry directed to the Keeper of the Printed Books at the British Museum in London brought the reply that in the work entitled, the *Construction and Use of the Line of Proportion*, London, 1628, the slide rule is explained. Prefixed to the book is a diagram of the "line of proportion," now called slide rule. Wingate says in his preface, "I have invented this tabular scale or line of proportion." Further on he says "the line of proportion is a double scale, broken off in tenne Fractions, upon which Logarithms of numbers are found out." This book was probably reproduced two years later in Wingate's work *Arithmetic made easy, or natural and artificial arithmetic*, London, 1630, a text quoted by Favaro* in his history of the slide rule. A second edition of Wingates publication of 1630 appeared in 1652, wherein improvements in the divisions of the slide rule are described. From these facts it appears that De Morgan was in error, and that the claim made for Edmund Wingate as the inventor of the straight-edge slide rule is well founded, for he published four years earlier than did William Oughtred. It should be added, however, that Oughtred describes also a circular slide rule and that he has a clear title as the inventor of the circular type.

My second point relates to the invention of the "runner." In 1850 a French artillery-officer and mathematician, A. Mannheim, designed a slide rule with a "runner," now generally known as the "Mannheim rule." German writers† have called attention to the fact that Mannheim was not the first inventor of the runner, that a description of it occurs in a French work of 1837, thirteen years earlier. My own reading reveals that the runner was invented much earlier in England and afterwards completely forgotten by the English. The first traces go back to Sir Isaac Newton, but in 1842 even De Morgan who writes at length on the slide rule and its history, makes no reference whatever to the runner. It is not generally known that Sir Isaac Newton referred to the slide rule. In Newton's works is given an extract from a letter of Oldenburg to Leibniz, dated June 24, 1675, which we shall consider more fully later. The "runner" is not mentioned in this extract, but Newton's slide rule could not be used without the employment of some device like that of the "runner." Sixty-eight years later, Newton's scheme slightly modified is explained more fully in Stone's *Mathematical Dictionary*, 2nd Ed., 1743. I am not aware that Newton's and Stone's slide rules were ever actually constructed and used in practice. But thirty-five years after Stone's publication a book was published in London, containing

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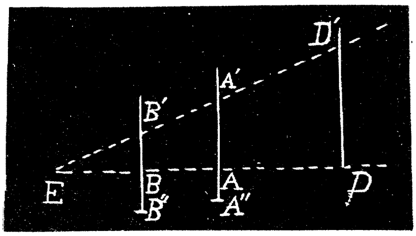
* *Veneto Instituto Atti* (5) 5, 1878-79, p. 495, abbreviated in Favaro's *Lecons de statique graphique*, 2 eme partie, *calcul graphique*, Paris, 1885, translated into the French by P. Terrier.

† *Zeitschrift f. Math. and Phys.*, Vol. 48, 1903, p. 134.

an instrument by John Robertson, which employed the runner and which was constructed in Cornhill by Messrs. Nairne and Blunt, and put upon the market. There are no indications that Robertson's rule ever became popular. Later the use of the runner was advocated by William Nicholson in an article printed in the Philosophical transactions of 1787. But in the first half of the nineteenth century I have not been able to find a single reference to the "runner" in England. It was completely forgotten.

Returning to Newton, I shall take up my third point, the early use of the slide rule in the solution of numerical equations. Oldenburg's letter to Leibniz, previously referred to, reads in translation from the Latin as follows: "Mr. Newton, with the help of logarithms graduated upon scales by placing them parallel at equal distances or with the help of concentric circles graduated in the same way, finds the roots of equations. In the arrangement of these rules all the respective coefficients lie in the same straight line. From a point of which line, as far removed from the first rule as the graduated scales are from one another, in turn, a straight line is drawn over them, so as to agree with the conditions conforming with the nature of the equation; in one of these rules is given the pure power of the required root."

If my interpretation of this passage is correct, it means in the case of the cubic $x^3+ax^2+bx=c$ that the rules A, B, D must be placed parallel and equidistant. On rule A find the number equal to the numerical value of the coefficient a; on rule B find the number equal to the numerical value of b, and on rule D find unity. Then arrange these three numbers on the rules in a straight line BD. Select the point E on this line, so that $BE=BA$. Through E pass a line ED' and turn it about E until the numbers at B', A', and D', with their proper algebraic signs attached, are seen to be together equal to the absolute term c.



Then the number on the scale D' is equal to $|\,x^3\,|$, and x can be found.

Remembering that the length of $B''B$ is $\log |\,b\,|$, and assuming $BB'=\log |\,x\,|$, it follows that $B''B'$ is equal to $\log |\,bx\,|$. Then $AA'=2\log |\,x\,|$, or $\log |\,x^2\,|$ and $A''A'=\log |\,ax^2\,|$, and $DD'=\log |\,a^3\,|$. The value of x can be found by moving the scale B up until B'' reaches the point B. The number on the scale at B' will then give the numerical value of the root. A device, as represented by the line ED', fulfills some of the functions of what is now called the "runner."

In Stone's *Dictionary* (1743) Newton's scheme is modified somewhat. Stone assumes that the equation is so transformed that all its coefficients, except the absolute term, are positive, The rules are contiguous and are not all graduated alike, but have, respectively, a single, double, triple, quadruple, etc., radius. This device calls for a runner of the type now in use, carrying a thread that is at right angles to the rules. Otherwise the gen-

eral plan for the numerical solution of equations is the same as with Newton.

As a fourth point in the history of the slide rule, I desire to point out that, while so generally known to writers on the slide rule, the English astronomer William Pearson was the first one to suggest, in 1797, the inversion of the slider for certain operations with the slide rule, the inversion of fixed lines on the slide rule had been introduced more than one hundred years earlier in Everard's slide rule, used in gauging.

Finally, I desire to say a word as to the introduction of the slide rule into the United States. Brief directions for the use of the slide rule appeared in a few arithmetics imported, or reprinted in this country, in the latter part of the eighteenth century. Thus, the *Arithmetic* of George Fisher, which is a pseudonym for Mrs. Slack, probably the first woman who is the author of a popular arithmetic, contained rules for the use of the slide rule. Her books were read in the United States. In Nicolas Pike's arithmetic, an American text of 1788, such rules were given. An edition of the English book, Dilworth's *Schoolmaster's Assistant,* was brought out in Philadelphia in 1805 by Robert Patterson, professor of mathematics in the University of Pennsylvania. It devotes half a dozen pages to the use of the slide rule in gauging. Another English work, Hawney's *Complete Measurer* (1st English Edition, 1717), was printed in Baltimore in 1813. It describes the English carpenter's rule, also an English rule for gauging. Of American works, Bowditch's *Navigator,* 1802, gives one page to the explanation of the slide rule, but when working examples, Gunter's line alone is used. From these data it is difficult to draw reliable conclusions as to the extent to which the slide rule was then actually used in the United States. We surmise that it was practically unknown. The Swiss geodesist, F. R. Hassler, who came to this country and became the first superintendent of the United States Coast and Geodetic Survey, is known to have used a slide rule. The present writer had the good fortune of inspecting Hassler's slide rule. But before 1880 or 1885 it is very difficult to find references to the slide rule in American engineering literature. I have seen a reference to the slide rule in a book issued in the first half of the last century by a professor of the Rensselaer Polytechnic Institute. From this institute was graduated in 1863 Mr. Edwin Thacher, a bridge engineer, who in 1881 patented his well-known cylindrical slide rule. Interest in slide rules was awakened about this time. It was in 1881 that Robert Riddell published in Philadelphia his booklet on *The Slide Rule Simplified.* In the preface he points out that, though nearly unknown in this country, the instrument was invented before the time when William Penn founded Philadelphia. But the slide rule never became really popular in the United States until the introduction of the Mannheim type. Keuffel and Esser imported Mannheim rules in 1888 and began the manufacture of them in Jersey City in 1895. An inquiry* instituted by C. A.

**Engineering News,* Vol. 45, 1901, p. 405.

Holden in 1901 showed that in about half of the engineering schools of the United States, attention is given to the use of the slide rule.

A BIQUADRATIC EQUATION CONNECTED WITH THE REDUCTION OF A QUADRATIC LOCUS.

By DR. ARTHUR C. LUNN, The University of Chicago.

If the equation of a conic section be written in the form

$$Ax^2+By^2+2Cxy+2Dx+2Ey+F=0,$$

then it is known that a rotation of the coordinate axes through an angle a will bring them into parallelism with the axes of symmetry of the curve, provided this angle is determined by

$$\tan 2a=\frac{2C}{A-B}.$$

This rotation corresponds to the substitution

$$(1) \qquad \begin{aligned} x&=x'\cos a-y'\sin a,\\ y&=y'\cos a+x'\sin a, \end{aligned}$$

with a so chosen as to eliminate the term in $x'y'$. But the sine and cosine may be expressed in terms of the tangent of the half-angle, thus:

$$(2) \qquad t=\tan\frac{a}{2}, \quad \cos a=\frac{1-t^2}{1+t^2}, \quad \sin a=\frac{2t}{1+t^2},$$

and the use of these in (1) gives the substitution expressed rationally in terms of the parameter t. Without reference to its trigonometric source, the substitution in that form is seen to be orthogonal or rotational for all values of t, since the equation of constancy of distances:

$$(x'_1-x'_2)^2+(y'_1-y'_2)^2=(x_1-x_2)^2+(y_1-y_2)^2,$$

is directly verifiable as an identity in t.

The use of this parameter makes it possible to effect the reduction of the conic by purely algebraic processes, independently of the trigonometric formulae. For the term in $x'y'$ will have as coefficient